

University of Information Technology and Sciences

Faculty of Science and Engineering

Department of CSE

Midterm Examination, Autumn- 2023

Course Title: Differential and Integral Calculus

Course Code: MAT 163

Marks: 20

Time: 1 hour

Answer all the questions

1. (a) Define even function and odd function. Test whether the following functions are even or odd. [3]

(i) $f(x) = \ln(x + \sqrt{1 + x^2})$

(ii) $f(x) = \frac{|x| - x^2}{2\cos x}$

- (b) Sketch the graph of the following functions. Also find domain and range of the following functions. [4]

(i) $f(x) = 2 + \sqrt{x - 4}$ (ii) $f(x) = 1 + |x - 2|$

- (c) If $f(x) = x^2 + 1$, $g(x) = \frac{1}{x}$ and $h(x) = x^3$, find $(f \circ g \circ h)(x)$. [3]

2. Find $\frac{dy}{dx}$. [10]

(i) $x^3 + y^3 = 3xy$ (ii) $\sin(x^2 y^2) = x$ (iii) $y = x^3 \sin^2(5x)$

(iv) $y = [1 + x^2 \sin^3(x^5)]^{12}$ (v) $y = \sqrt{x^3 + \operatorname{cosec} x}$

$$\cot x = -\operatorname{cosec}^2 x$$

$$\sec x = \sec x \tan x$$

$$\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

Marks: 20

Time: 1(one) hour

(Answer all questions)

1. (a) Find domain and range of the following functions and also sketch the graph of the following functions : [03]

(i) $f(x) = \sqrt{-x-2} - 4$ (ii) $f(x) = \frac{x}{x+3}$

- (b) Define even function and odd function. Test whether the following functions are even or odd. [04]

(i) $f(x) = (x + \sqrt{1+x^2})$

(ii) $f(x) = \frac{\tan x}{x + \sin x}$

- (c) If $f(x) = \sqrt{x^3 + 2\sqrt{x}}$, $g(x) = (1+x)^{-1}$ and $h(x) = x^{3/2}$, find $(f \circ g \circ h)(x)$. [03]

2. (a) A function $f(x)$ is defined as follows. [02]

$$f(x) = \begin{cases} 2x + 3, & x \leq 4 \\ 7 + \frac{16}{x}, & x > 4 \end{cases}$$

Discuss the continuity of $f(x)$ at $x = 4$.

- (b) Find $\frac{dy}{dx}$. [08]

(i) $x^3y + 4xy^2 = 3xy$ (ii) $\sin(x^2y^2) = x$

(iii) $y = \frac{\sin x}{1 + \cos x}$ (iv) $y = [1 + \cos^3(\sin 2x)]^{-3}$